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Robust Design Optimization by First Design Method

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Launch vehicles are subjected to severe vibratory conditions during takeoff and flight ascent. The loads are not well known at the very beginning of a project, and, even at the end, some uncertainties still prevail. In spite of this, the preliminary design has to be robust with regard to the vibration levels encountered at the payload/launch vehicle interface. This paper presents a new robust design methodology called the First Design Method. This methodology has been applied to a typical launch vehicle excited by a low-frequency transient load. The First Design Method optimizes the natural frequencies of the structural configuration of interest and, subsequently, designs the structural configuration with the modes provided by the optimization procedure. Furthermore, the method, which is based on energy conservation for integrating uncertainties about excitation, is compliant with the laws of physics. In the industrial case investigated in this paper, the First Design Method has been used to design a typical launch vehicle with a very satisfactory behavior as far as the mass and vibration levels at the payload/launch vehicle interface are concerned.

Nomenclature

a_k	=	Fourier coefficients corresponding to the cosine terms
b_k	=	Fourier coefficients corresponding to the sine terms
$\mathbf{c}$	=	vector of the Fourier coefficients
d	=	distance from the optimal set of natural frequencies
E	=	energy of the excitation
$f(t)$	=	excitation signal related to time
f_i	=	frequency of the ith natural mode
$f_{i\,\mathrm{opt}}$	=	frequency of the ith optimized natural mode
H_i	=	matrix representing the transfer function
$H_i(\omega)$	=	transfer function of a 1-degree-of-freedom system
N	=	number of elements of the vector defining the excitation
p	=	number of frequencies taken into account in the Fourier series
p_c	=	crossover probability by bit
p_m	=	mutation probability by bit
T_e	=	time step of the vector defining the excitation
X_i	=	ith design parameter
$X_{i\,\mathrm{opt}}$	=	optimized ith design parameter
$x_{4000}(t)$	=	horizontal displacement at node 4000 related to time
$\mathbf{z_i(t)}$	=	vector of the responses to the harmonic excitations of the Fourier series
μ_{4000}, μ_{3203}	=	vibratory levels at nodes 4000 and 3203 of the finite element model
ξ_i	=	modal damping of the ith natural mode
σ_{4000}, σ_{3203}	=	standard deviations of the vibratory levels at nodes 4000 and 3203 of the finite element model
ϕ_{ix}	=	horizontal component of the ith eigenvector
$(\omega_{0i}, \boldsymbol{\sigma}_\mathbf{i})$	=	ith natural mode of the structure (natural frequency and eigenvector)
$(\omega_{0i}, \boldsymbol{\sigma}_\mathbf{i})_{\mathrm{opt}}$	=	optimized ith natural mode

I. Introduction

FUNDAMENTALLY, robust design is concerned with minimizing the effect of uncertainty or variation in design parameters on a design without eliminating the source of uncertainty or variation. In other words, a robust design is "less sensitive" to variation in uncontrollable design parameters than the traditional optimal design point. This design approach is necessary because many real systems from all fields of engineering are affected by random and uncontrollable inputs. One common cause of uncertainty occurs as a result of uncontrollable probabilistic (random) deviations in the values of the design parameters. These deviations may arise from many sources, including changes in environmental conditions, variation in material composition, or manufacturing variations.

In recent years, a number of approaches have been proposed in the literature to attain robust designs. These include Taguchi's "parameter design concept" [1–3], which was developed to minimize variations in performance caused by variations in noise factors (uncontrollable parameters). Basically, in this method, a two-part orthogonal array is used for experimental design using the signal-to-noise ratio as an optimization criterion. Although Taguchi's contribution is recognized for facilitating optimization for engineers who have very little training in optimization methods, there are certain assumptions and limitations associated with this method. For example, use of the Taguchi method will not yield an accurate solution for design problems involving highly nonlinear behavior. This is why many other approaches have been developed.

For single-objective robustness, there are two types of robust optimization approaches described in the literature: deterministic and probabilistic [4]. Deterministic approaches provide an objectively robust optimum design by analytically measuring the robustness of a design alternative using its first-order derivative or other nonstatistical measures and then incorporating these measures

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into the approach. Probabilistic approaches, on the other hand, use statistical concepts to measure the robustness of a design and then optimize the design based on that information. The two most commonly used statistical measures are the mean and variance of the objective values. Thus, some approaches in the literature address the issue of multi-objective robustness, but the concept of multiple-objective functions is not yet widely developed.

In evolutionary algorithms, a number of prominent new studies have been undertaken in recent years on the handling of uncertainty in engineering designs to which the Monte Carlo techniques can be applied to evaluate both the mean and variance of a response. These techniques, which are theoretically simple, have the disadvantage of being very time consuming; in fact, to obtain statistically reliable results, it is necessary to investigate hundreds of cases, even thousands. The larger the number of design parameters, the larger the number of cases to be computed. An advantage of this method is that it enables a visualization of the influence of the different parameters, and so it provides guidelines for the best design, which could not have been analyzed using a classical optimization procedure.

The main principles of evolutionary algorithms come from human genetics, as does the vocabulary. They are implemented as a computer simulation in which a population of abstract representations (called chromosomes, genotype, or genome) of proposed solutions (called individuals, creatures, or phenotypes) to the optimization problem evolves toward better solutions [5]. Traditionally, solutions are represented in binary form as strings of zeros and ones, but other encodings are also possible. The evolution usually starts from a population of randomly generated individuals and leads to generations. In each generation, the fitness of every individual in the population is evaluated and then multiple individuals are stochastically selected from the current population (based on their fitness) and modified (recombined and possibly randomly mutated) to form a new population. The new population is then used in the next iteration of the algorithm. Thus, different key steps are necessary to achieve the genetic algorithms: encoding the parameters, generating the initial population, evaluating, selecting, reproducing (mutation and crossover), and checking a criterion of termination.

It can be noticed that the use of these algorithms is based on the assumption that the best solutions will be found in regions of the search space containing relatively high proportions of good solutions.

These algorithms seem particularly desirable for solving multi-objective optimization problems because they deal simultaneously with a set of possible solutions (the so-called population), which allows the identification of several members of the Pareto optimal set in a single run of the algorithm instead of having to perform a series of separate runs, as in the case of the traditional mathematical programming techniques. Additionally, evolutionary algorithms are less susceptible to the shape or continuity of the Pareto front (for example, they can easily deal with discontinuous or concave Pareto fronts).

The objective of this paper is to present a new robust design procedure called the First Design Method (FDM) [6–8]. The main issue will be to reduce the low-frequency vibration levels encountered by payloads or equipment mounted on the launch vehicle when it is subject to a transient load, for example. Furthermore, the excitation is not completely known. The development of a robust design methodology will help improve the robustness of the product while minimizing the effects on the response of the excitation uncertainties. This new robust optimization procedure tackles the intrinsic characteristics of the structure because it is based on the optimization of the natural modes of the structure thanks to Monte Carlo simulations (probabilistic measures are evaluated and used to obtain a robust optimum). A computation of the response by modal superposition is used, introducing the uncertainties of the excitation on its Fourier coefficients. Finally, the material description of the structure including its geometrical parameters is obtained using Monte Carlo simulations or a genetic algorithm.

II. Presentation of the Study Case

A. Model

This paper aims to set up a procedure applicable to an industrial case. The case of a small launcher, fictitious but representative of launchers currently on the market, with a simplified low-frequency model will be studied. The numerical model of the launch vehicle has been simplified to easily verify the efficiency of the FDM. The payload is modeled, representative of real payloads (in terms of mass, damping, stiffness, and frequencies). Thus, the model has been tuned so that its natural modes are typical of existing launchers (frequencies, mode shapes, deformation, and kinetic energies).

The numerical model, which is performed with Nastran, is shown in Fig. 1; it is bidimensional and composed of three propulsive stages and a fairing with a payload. The boosters are modeled by punctual masses positioned at the height of the gravity centers of the boosters. The beam elements that compose the structure are made of aluminum with tube sections. The three masses are linked to these beam elements by spring-damper systems. Thus, the many parameters of these different parts of the model have been tuned so that the natural modes of the system are typical of those of industrial launchers.

B. Optimization Problem

A particular excitation is applied to the numerical model: a transverse transient load, representative of a sudden gust of wind. This excitation is not completely known, so the design of the launch vehicle must be robust with regard to the excitation.

Furthermore, acceptance and qualification vibration levels are specified for equipment and payloads. Among the specifications is a limit value for the vibration level at the payload/launch vehicle interface. Electronic equipment can also be sensitive to these vibrations encountered during flight ascent.

Then the optimization is performed to minimize the mass of the system; the design parameters here, which are directly controlled by the designer and shown in Fig. 2, are four different thicknesses. The constraints are the specifications previously defined.

III. Taguchi's Approach

The Taguchi method was developed in the late 1940s to improve quality [1,2]. According to Taguchi's principles about robust design, the design parameters can be classified into three categories: design factors (controlled by the designer), noise factors (uncertain), and responses (which characterize the system's performance). The aim of robust design is to reduce the variations in a system's performance caused by the uncertain values of noise factors and, thus, to reduce the sensitivity of the system. Therefore, the proposed solutions try

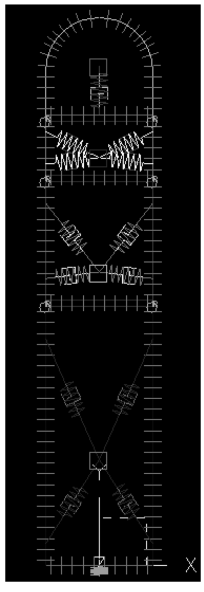

Fig. 1 Finite element model of the launcher.

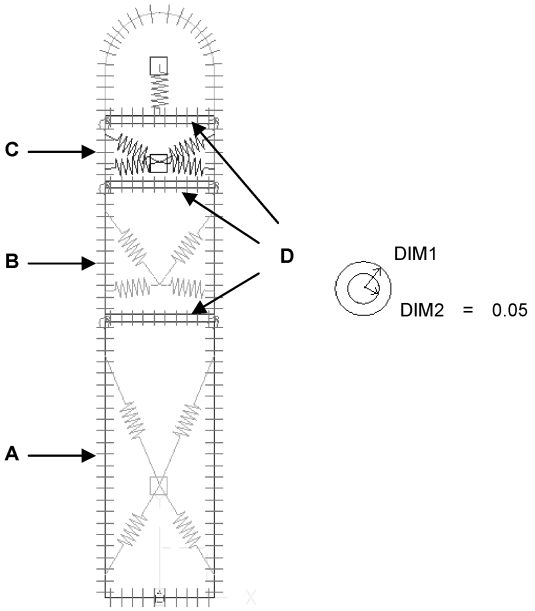

Fig. 2 Design parameters of the model.

to minimize the response variations while reaching performance targets.

Taguchi's approach is based on the use of plans of experiments. The estimates of average performance and performance variations are obtained thanks to a plan of experiments created by crossing two matrices: an inner array, representing the design factors, and an outer array, representing the noise factors. It enables an estimation of the means and variances of the responses. Then, the different design factor combinations can be compared using evaluation criteria; those used by Taguchi to estimate the performance and the minimization of sensitivity are the *signal-to-noise ratio* or the *loss function* [3,9].

To perform an optimization taking into account the system response sensitivity, a procedure based on Taguchi's principles has been developed. Once the finite element (FE) model has been defined, the main steps of the procedure will be the following: identification of the design and noise factors and of the responses; definition of the search space; definition of the variation spaces of the noise factors (intervals, distributions, etc.); postulation of a response function, that is, responses related to design factors (linear, quadratic, with interactions, etc.); definition of plans of experiments for design factors and noise factors (number of levels for each factor, number of simulations to perform, etc.); computation of the simulations; calculation of the approximate model parameters; formulation and resolution of the optimization problem thanks to the analytical approximation of the response function; and, finally, checking that the FE model provides the same responses for the optimum computed design factors as the ones predicted by the approximate models.

This procedure was performed on a given launcher subject to transient excitation. The design factors are the same as the ones described previously, that is, the four thicknesses shown in Fig. 2 and the three responses, which are the mass, the average vibratory level at the payload/launch vehicle interface, and its standard deviation. The noise factors are defined as multiplying factors applied to the magnitude and the duration of the excitation. The postulated response models are quadratic with interactions:

$$\hat{y} = c_0 + c_1A + c_2B + c_3C + c_4D + c_{11}A^2 + c_{22}B^2 + c_{33}C^2 + c_{44}D^2 + c_{12}AB + c_{13}AC + c_{14}AD + c_{23}BC + c_{24}BD + c_{34}CD + c_{123}ABC + c_{124}ABD + c_{134}ACD + c_{234}BCD \quad (1)$$

Thus, three levels in the plan of experiments are necessary for the design factors. The plan used is shown in Table 1. The inner array is a central composite plan. This kind of plan is widely used to obtain approximate models as response surfaces.

The coefficients of the approximate models are calculated using the results of the simulations and the least-squares technique.

For each of the three responses, an approximation is given by

$$\hat{\mathbf{y}} = X\hat{\mathbf{c}} \quad (2)$$

where X is the matrix of experiments, and $\hat{c}$ is the vector of the model coefficients:

$$\hat{\mathbf{c}} = (X^TX)^{-1}X^T\mathbf{y} \quad (3)$$

where $\mathbf{y}$ is the vector of the computed responses.

This technique makes it possible to obtain approximate models for which the accuracy is particularly satisfactory given that the search space for the design factors is not very large.

Thus, the optimization is performed on the approximate models using the MATLAB® function fmincon and the following problem formulation:

$$\begin{cases} \text{minimize the mass of the system} \\ \text{subject to } \mu_{4000} + k.\sigma_{4000} < \text{limit} \end{cases}$$

where μ_{4000} is the mean of the vibration level at the payload/launch vehicle interface, and σ_{4000} is the standard deviation at this same point.

Table 2 shows the results with the variation spaces of the different factors shown in Table 3 for $k = 3$ and for different values of the limit.

It should be noted that these results, in the case of a reduced search space, are very satisfactory. This procedure makes it possible to provide efficient structures for which the predicted behaviors are close to the ones computed by the Nastran simulations. However, the method is efficient because our FE model is quite simple; this will not provide an accurate solution for industrial design problems that often involve highly nonlinear behavior, because the responses of such systems would not be accurately approximated by polynomial functions of the design factors. Moreover, this way of tackling the

Table 1 Plan of experiments used

Magnitude	−1	−1	−1	0	0	0	1	1	1
Duration	−1	0	1	−1	0	1	−1	0	1

No.	A	B	C	D
1	−1	−1	−1	−1
2	1	−1	−1	−1
3	−1	1	−1	−1
4	1	1	−1	−1
5	−1	−1	1	−1
6	1	−1	1	−1
7	−1	1	1	−1
8	1	1	1	−1
9	−1	−1	−1	1
10	1	−1	−1	1
11	−1	1	−1	1
12	1	1	−1	1
13	−1	−1	1	1
14	1	−1	1	1
15	−1	1	1	1
16	1	1	1	1
17	−1	0	0	0
18	1	0	0	0
19	0	−1	0	0
20	0	1	0	0
21	0	0	−1	0
22	0	0	1	0
23	0	0	0	−1
24	0	0	0	1
25	0	0	0	0

Table 2 Optimization results

$k = 3$ (99.865% of reliability)					
Limit, $m \cdot s^{-2}$	A, m	B, m	C, m	D,m	Mass, kg
195	0.218787	0.18	0.22	0.090642	60765.69
200	0.207386	0.18	0.22	0.084563	60158.19
205	0.197764	0.18	0.22	0.08	59681.73
210	0.188127	0.18	0.22	0.08	59303.23
215	0.18	0.18	0.219649	0.08	58995.41
220	0.18	0.18	0.197265	0.08	58796.67

	Predicted values, $m \cdot s^{-2}$ (provided by the approximate models)			Correct values, $m \cdot s^{-2}$ (provided by Nastran simulation)		
Limit, $m \cdot s^{-2}$	Average level	Standard deviation	Mean + k. std deviation	Average level	Standard deviation	Mean + k. std deviation
195	180.83	4.72	195	180.87	4.78	195.19
200	183.72	5.43	200	183.09	6.47	202.52
205	187.2	5.93	205	187.56	5.95	205.42
210	191.27	6.24	210	191.94	5.99	209.91
215	195.34	6.55	215	195.3	6.47	214.71
220	199.76	6.75	220	199.71	6.49	219.18

Table 3 Variation spaces of the different factors

	Level	Value
A, B, and C	−1	0.18 m
	0	0.2 m
	1	0.22 m
D	−1	0.08 m
	0	0.1 m
	1	0.12 m
Magnitude	−1	0.975
	0	1
	1	1.025
Duration	−1	0.475 s
	0	0.5 s
	1	0.525 s

robust optimization issue has some disadvantages. First, the search space is limited; this robust optimization method will be efficient to optimize a system. However, it only allows the designer to find a structural configuration close to the nominal one and, therefore, to achieve a local robust optimization. Furthermore, the way in which uncertainties on the excitation are taken into account is quite simplistic because it merely consists of multiplying factors applied to the magnitude and the duration of the excitation.

IV. Frame of Reference

A. *V*-Cycle Context

Systems engineering today is the methodological framework that allows the design of complex mechanical systems to meet specifications with maximum reliability. The proposed method has been successfully applied to the optimization of the dynamics of a vehicle [6,7]. The basic idea is a hierarchical organization with functional sets, subsystems, and parts, and definitions of the technical specifications for each design level. In the *V*-cycle context, systems engineering is an optimization procedure based on a hierarchical organization of the parameters, assuming that the choice made at a level of the cycle does not call into question any previous choices. Two stages can be distinguished: the descent of the cycle, associated with the product design; and the ascent of the cycle, associated with its integration with different tests. This cannot be done scientifically without conceptualizing the architecture of the solutions. Two design stages are therefore clearly defined and shown in Fig. 3, characterized on the one hand by the functional architecture and on the other hand by the material architecture.

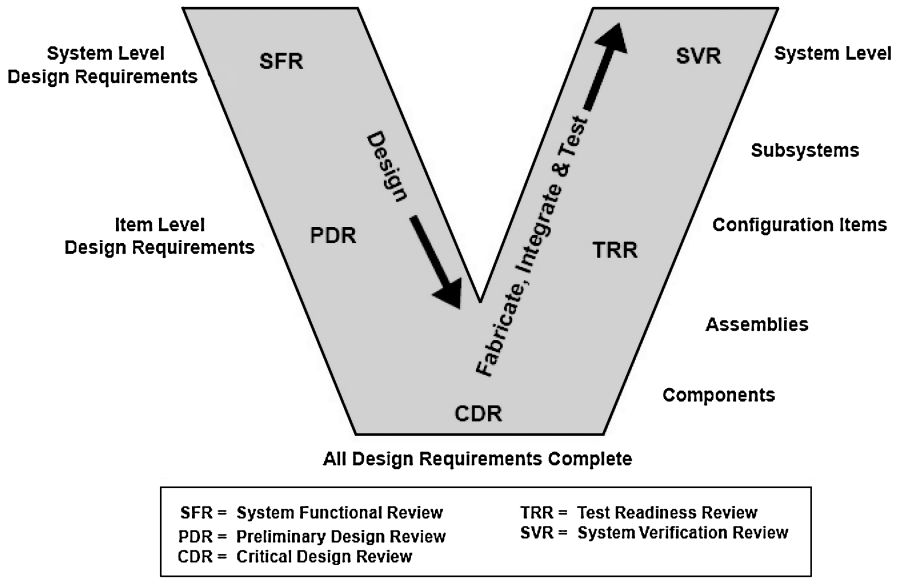

Fig. 3 V-cycle scheme.

B. Main Principles of the First Design Method

The first purpose of the design stage is to determine the values of the design parameters that satisfy the many constraints, requirements, and specifications of the product with reasonable safety margins. Each objective can be optimized separately in the design space, but the optimization procedure consists of finding the best compromise to solve a multi-objective optimization problem. In this context, the Pareto set of solutions is a convenient and commonly used tool but, in practice, the ideal that would provide the simultaneous satisfaction of each specification is very difficult to obtain. More precisely, the defined objectives are often antagonistic, which is why a compromise is required.

Furthermore, during the design phase, the designer will have to deal with uncertainties. Instead of taking them into account after the design, a solution is to integrate them in the optimization procedure to find a solution that would offer both performance and low sensitivity to variations. This is the main idea of robust design [6–8]. Various kinds of uncertainties may be encountered: ones concerning system parameters (length, stiffness, etc., which are, for example, linked to manufacturing tolerances), and others concerning the characteristics of the external environment (external conditions, loads, other systems, etc.).

This approach to system optimization, based on the notion of quality, introduces the concept of robustness. It may often be preferable to find a solution with lower performance, but that is less sensitive to uncertainties. Thus, it is especially important to modify the optimization procedures by introducing the notion of a "robust solution." Because reduced sensitivity (or robustness) is often in conflict with performance, the compromise will be a solution with lower performance than the one computed by a simple optimization, but that is less sensitive.

In our case, which concerns low-frequency vibrations, the functional analysis proposed by the FDM [7,8] will be performed on the natural modes of the structure. The modal synthesis will constitute our functional model whereas the FE model will be the material model. This is why the optimization will primarily be performed on the natural modes before going back to the geometry with the FE model afterward. The basic architecture of the optimization procedure is represented in Fig. 4.

V. Description of the First Design Method

A. Taking into Account Uncertainties

The paper aims to introduce a robustness performance in relation to the excitation uncertainties, and so it is necessary to describe the tools used to take these uncertainties into account. Convex analysis is one of the tools [10,11]. In fact, there are cases in which the uncertain parameters can be reduced to convex sets. Thus, the resolution of a convex analysis problem is the resolution of an optimization problem subject to constraints that consist of finding the extreme values of the system response, knowing that the input parameters are in a convex set. The interesting aspect is that it allows the determination of "conclusive" solution sets, that is, containing 100% of the possible values of the solution parameters; this can prove very interesting in terms of reliability because it can make possible a "zero defect" solution. However, the results risk being "too conservative" in the sense that the structure may be oversized if the extreme limits obtained are taken into account. To reduce this problem, the designer must pay particular attention to the definition of the convex set containing the possible values of the uncertain parameters.

This is why the probabilistic methods are the ones most commonly used to deal with uncertainties. They consist of taking into account a probability law for each uncertain input parameter; this makes it possible to obtain the probability law of the response.

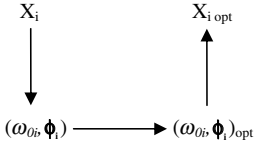

Fig. 4 Architecture of the optimization procedure.

In our case, the structure is considered deterministic, that is, known without uncertainties. The excitation will be introduced in the optimization procedure with uncertainties. These uncertainties will be applied to the Fourier coefficient of the excitation according to [12].

The excitation, as energy, is perfectly known in the nominal case. This energy is considered to be constant:

$$\int_0^{NT_e} f^2(t)\,\mathrm{d}t = E \tag{4}$$

This condition gives us a relation between the Fourier coefficients:

$$\int_0^{NT_e} f^2(t)\,\mathrm{d}t = \int_0^{NT_e} \left(\sum a_k \cos\left(\frac{2\pi k}{NT_e}t\right) + b_k \sin\left(\frac{2\pi k}{NT_e}t\right)\right)^2 \mathrm{d}t = \frac{NT_e}{2}\sum a_k^2 + b_k^2 \tag{5}$$

Then, the first condition is

$$\sum a_k^2 + b_k^2 = \frac{2E}{NT_e} \tag{6}$$

Furthermore, the problem is solved by computing the Fourier transform series of the excitation. Thus, the excitation is known as a finite sum of harmonics. In the given case, a satisfactory approximation of the load is obtained by using 40 terms of the Fourier transform series (the magnitudes of the harmonics of the excitation are given in Fig. 5). Then, the uncertainties are introduced in the 80 Fourier coefficients (sine and cosine terms for each frequency). They will describe normal laws around the nominal spectrum.

It is necessary to define the numerical values of the standard deviations of these normal laws. With the measures conducted in a defined period of time, it is assumed that knowledge of the high-frequency terms is better than knowledge of the low-frequency terms. Thus, the standard deviations will depend on the frequency; they will be larger for the first frequencies than for the others. For the first 12 frequencies, $\sigma = 0.58 \times$ nominal coefficient/3; for the 13 following frequencies, $\sigma = 0.41 \times$ nominal coefficient/3; and, finally, for the last 15 frequencies, $\sigma = 0.26 \times$ nominal coefficient/3.

These values are justified by the fact that a 2 dB rise is a multiplication by 1.26, 4 dB by 1.41, and 6 dB by 1.58. Then, the standard deviations are divided by three, so that 99.73% of the coefficients are in the intervals for which the bounds are

$$\text{nominal coefficient} \pm \Delta c \tag{7}$$

where Δc is successively 58, 41, and 26% of the nominal coefficient.

Finally, a corrective factor is applied to the Fourier coefficients, which are randomly drawn following the corresponding normal laws to keep the sum of the squares of the Fourier coefficients constant. This factor is

$$\sqrt{\frac{\sum \text{nominal coefficients}^2}{\sum \text{drawn coefficients}^2}} \tag{8}$$

In this way, the sum of the squares of the new coefficient will be equal to the sum of the squares of the nominal coefficients, according to the first condition (6).

B. Main Steps of the Procedure

The following main steps are performed in the framework of this method. First, it is necessary to compute the Fourier transform series of the excitation. It is then possible to compute the response of the

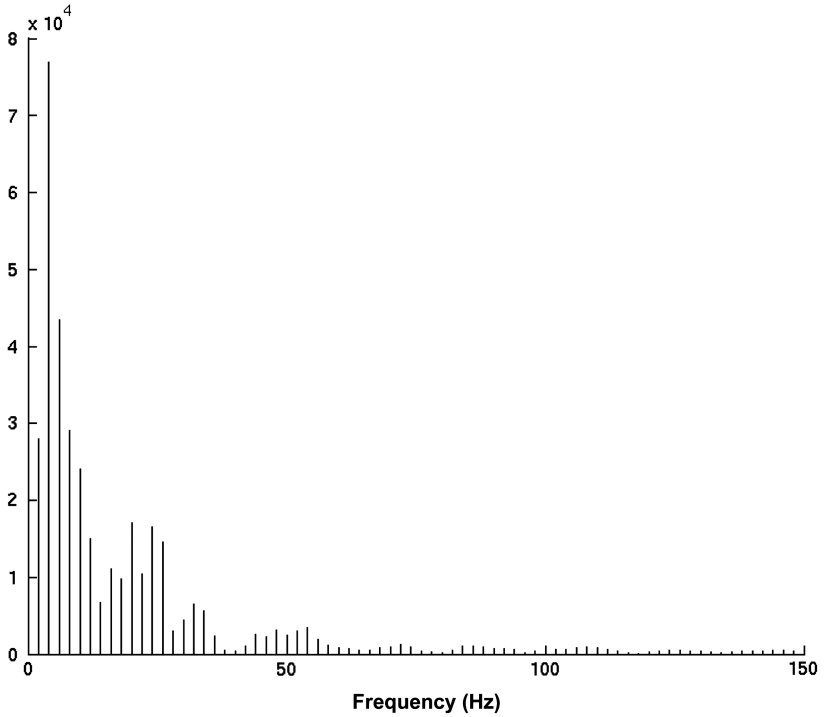

Fig. 5 Magnitudes of the harmonics of the excitation.

system to our excitation case as a weighted sum of responses to harmonic excitations. More precisely, for these harmonic excitations, the computation will be performed analytically by modal superposition associated with a functional overview of the mechanical systems. In this way, thanks to the reduction in computing time, many computations will be performed to optimize the natural frequencies of the mechanical system. Finally, it is necessary to go back to the geometry of the structure and design a configuration for which the natural modes are those provided by the optimization procedure. The main steps of the procedure are thus as follows.

Step 1: Initialization. The first step of the procedure is the computation of the Fourier transform series of the excitation, the definition of the variation spaces of the Fourier coefficients of the excitation, the programming of the modal superposition (MATLAB®) to calculate the response, and then the definition of the variation spaces of the natural frequencies and of the eigenvectors $(\omega_{0i}, \boldsymbol{\sigma}_\mathbf{i})$.

Step 2: Formulation of the optimization problem. The problem to be solved is the following: find $(\omega_{0i}, \boldsymbol{\sigma}_\mathbf{i})$ to minimize an objective function, taking into account the acceleration level at the payload/launch vehicle interface (with a limit value), but also the sensitivity of this level to the excitation variations. For example, the objective function can be defined as follows:

$$\mu_{4000} + k.\sigma_{4000} \tag{9}$$

where μ_{4000} is the averaged vibration level at the payload/launch vehicle interface, σ_{4000} is the standard deviation of this level, and k is a constant chosen by the designer. The choice of the value of k in this formulation, assuming that the performance variations are distributed according to a normal law, gives us the percentage of reliability (see Table 4), that is, the percentage of excitation cases in which the vibration level will be lower than the value of the objective function.

Step 3: Iterations. The Monte Carlo simulations of $(\omega_{0i}, \boldsymbol{\sigma}_\mathbf{i})$ using the MATLAB® modal superposition script are computed to find the best $(\omega_{0i}, \boldsymbol{\sigma}_\mathbf{i})$ according to the problem formulation. For each mode set, a high number of excitation cases must be analyzed to estimate the sensitivity of the response.

Step 4: Optimization of the system geometry. The design parameters minimizing the mass of the structural configuration must be found so that the natural frequencies are ω_{0i} and the eigenvectors are $\boldsymbol{\sigma}_\mathbf{i}$. This is done using Monte Carlo simulations or a genetic algorithm.

Table 4 Reliability percentages associated with the normal law

Level of sigma	Reliability level (bilateral constraint)	Reliability level (unilateral constraint)
±1	68.27	84.13
±2	95.45	97.725
±3	99.73	99.865
±4	99.9937	99.9968
±5	99.999943	99.999971
±6	99.99999980	99.99999988

C. Modal Superposition

Expressing the excitation as a sum of harmonic loads makes it possible to treat the resolution as a sum of simple responses.

Thus, the displacement at the base of the payload (node 4000 of the FE model) is computed as follows:

$$x_{4000}(t) = \left(\sum_{i=1}^{\text{nb of modes}} \mathbf{z_i^T(t)} H_i^T \right) \mathbf{c} \tag{10}$$

where c is the Fourier coefficients vector (if p frequencies are taken into account in the Fourier series):

$$\mathbf{c} = [\, a_1 \quad \cdots \quad a_p \quad b_1 \quad \cdots \quad b_p \,]^T \tag{11}$$

Furthermore, for each mode i taken into account in the modal superposition, H_i is the matrix representing the transfer function and $\mathbf{z_i(t)}$ corresponds to the vector of the responses to the harmonic excitations of the Fourier series:

$$\mathbf{z_i(t)} = \begin{bmatrix} A_i(\cos(2\pi f_1 t) - e^{-\xi_i\omega_{0i}t}\cos(\omega_{0i}\sqrt{1-\xi_i^2}t)) \\ \vdots \\ A_i(\cos(2\pi f_p t) - e^{-\xi_i\omega_{0i}t}\cos(\omega_{0i}\sqrt{1-\xi_i^2}t)) \\ A_i(\sin(2\pi f_1 t) - e^{-\xi_i\omega_{0i}t}\frac{2\pi f_1}{\omega_{0i}\sqrt{1-\xi_i^2}}\sin(\omega_{0i}\sqrt{1-\xi_i^2}t)) \\ \vdots \\ A_i(\sin(2\pi f_p t) - e^{-\xi_i\omega_{0i}t}\frac{2\pi f_p}{\omega_{0i}\sqrt{1-\xi_i^2}}\sin(\omega_{0i}\sqrt{1-\xi_i^2}t)) \end{bmatrix} \tag{12}$$

where

$$A_i = \phi_{ix4000} \sum_{\text{excitation nodes}} \phi_{ix}$$

Finally, the following can be obtained:

$$H_i = \begin{bmatrix} \mathrm{Re}(H_i(\omega_1)) & 0 & 0 & -\mathrm{Im}(H_i(\omega_1)) & 0 & 0 \\ 0 & \ddots & 0 & 0 & \ddots & 0 \\ 0 & 0 & \mathrm{Re}(H_i(\omega_p)) & 0 & 0 & -\mathrm{Im}(H_i(\omega_p)) \\ \mathrm{Im}(H_i(\omega_1)) & 0 & 0 & \mathrm{Re}(H_i(\omega_1)) & 0 & 0 \\ 0 & \ddots & 0 & 0 & \ddots & 0 \\ 0 & 0 & \mathrm{Im}(H_i(\omega_p)) & 0 & 0 & \mathrm{Re}(H_i(\omega_p)) \end{bmatrix} \tag{13}$$

where $\omega_k = 2\pi f_k = (2\pi k/NT_e)$, and $H_i(\omega)$ defined by

$$H_i(\omega) = \frac{1}{\omega_{0i}^2 - \omega^2 + 2j\xi_i\omega_{0i}\omega} \tag{14}$$

It can be noted that, concerning the calculation of the acceleration, two possibilities exist. First, having computed the displacement, the acceleration can be obtained by an approximation using the finite differences method:

$$\left.\frac{\partial^2 x}{\partial t^2}\right|_{x_i} \approx \frac{x_{i+1} - 2x_i + x_{i-1}}{T_e^2} \tag{15}$$

The same kind of calculation used for the displacement can also be performed by replacing the matrices H_i by

$$\begin{bmatrix} -\omega_1^2\mathrm{Re}(H_i(\omega_1)) & 0 & 0 & \omega_1^2\mathrm{Im}(H_i(\omega_1)) & 0 & 0 \\ 0 & \ddots & 0 & 0 & \ddots & 0 \\ 0 & 0 & -\omega_p^2\mathrm{Re}(H_i(\omega_p)) & 0 & 0 & \omega_p^2\mathrm{Im}(H_i(\omega_p)) \\ -\omega_1^2\mathrm{Im}(H_i(\omega_1)) & 0 & 0 & -\omega_1^2\mathrm{Re}(H_i(\omega_1)) & 0 & 0 \\ 0 & \ddots & 0 & 0 & \ddots & 0 \\ 0 & 0 & -\omega_p^2\mathrm{Im}(H_i(\omega_p)) & 0 & 0 & -\omega_p^2\mathrm{Re}(H_i(\omega_p)) \end{bmatrix} \tag{16}$$

The calculated system response is implemented on MATLAB® and provides results similar to those computed by a Nastran modal simulation (displacement and acceleration are shown in Figs. 6 and 7). Of course, the Nastran and the MATLAB® calculations must be performed with the same parameters, that is, the same modal damping and number of modes taken into account.

D. Looking for the Optimal Set of Natural Frequencies

The optimization is performed on the natural frequencies of the structure. Using the MATLAB® calculation by modal superposition and the computer time reduction, this optimization is achieved with Monte Carlo simulations. A random sampling of the frequencies is carried out and the best set is selected in accordance with the optimization problem formulation. In practical terms, the optimization is performed on a finite number of natural frequencies; thus, it is important to determine the number of modes needed to compute the response of the given structural configuration by modal superposition. In our case, six modes are necessary; consequently, the optimization of the six corresponding frequencies is performed.

However, before computing the Monte Carlo simulations, it is necessary to determine the values of the frequencies, the modifications of which are industrially possible with acceptable modifications of the design parameters. This is why it is essential to compute some Nastran modal simulations to test the influence of design parameter variations on the selected natural frequencies. In this way, the functional space is opened before limiting it and determining the best individual in the space. The choice of these prior simulations can be made with a small plan of experiments (small numbers of experiments and small numbers of levels for each design parameter), which could lead to an optimal exploration of the search space. Having computed these simulations, variation spaces for each frequency can be defined.

Then the difficulty in achieving the Monte Carlo simulations lies in the variations of the natural frequencies and of the eigenvectors. In fact, the values of the six selected frequencies are selected from the search spaces obtained, but these variations result in the modification of the eigenvectors and it has been seen before that the components of these vectors corresponding to the excitation nodes and to the node at which the response is measured play a role in the calculation of the response. This calculation of the components of the eigenvectors from the modification of the natural frequencies is done by obtaining approximate models. Using the results of the modal calculations of three different geometrical configurations, the constant vectors a_i and b_i can be obtained such that

$$\boldsymbol{\sigma}_\mathbf{i} = \mathbf{a_i}(f_i - f_{0i})^2 + \mathbf{b_i}(f_i - f_{0i}) + \boldsymbol{\sigma}_\mathbf{0i} \tag{17}$$

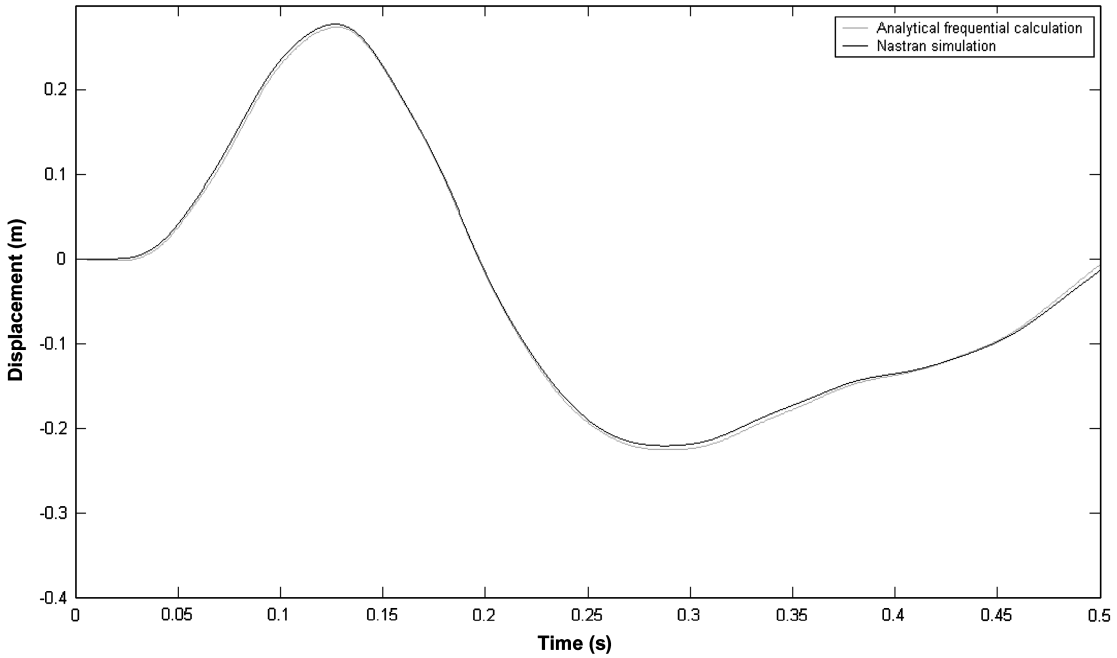

Fig. 6 Comparison of MATLAB® and Nastran displacements; modal superposition (20 modes), modal damping: 0.15.

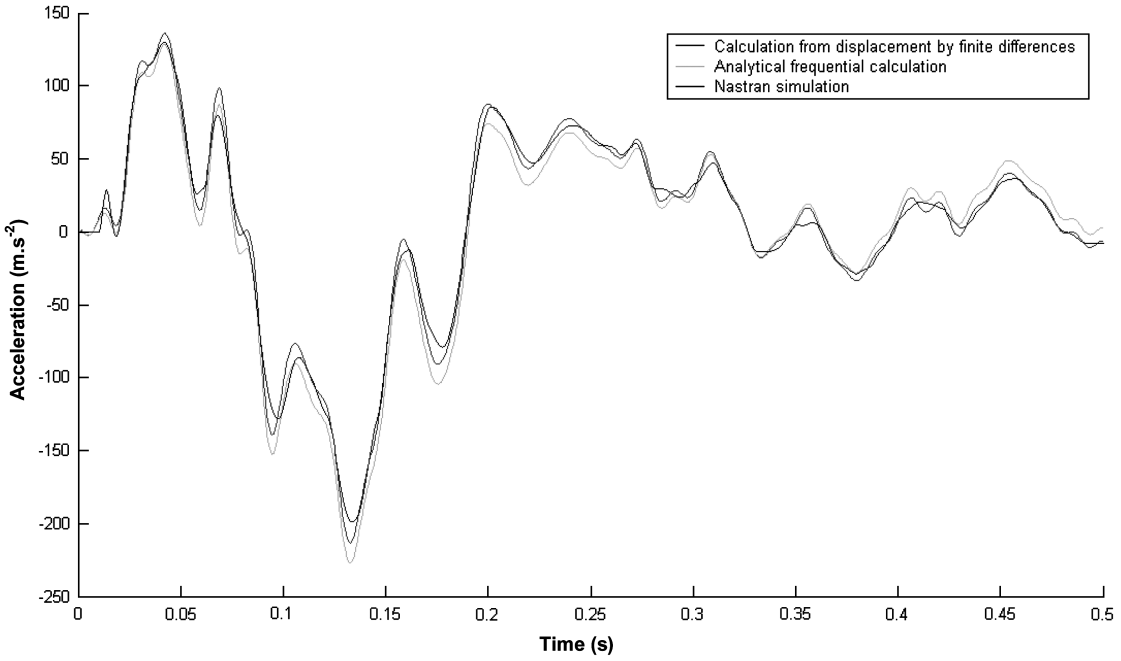

Fig. 7 Comparison of MATLAB® and Nastran accelerations; modal superposition (20 modes), modal damping: 0.15.

where i corresponds to the natural mode i and zero to the initial configuration of the structure, and $\phi_{\mathbf{i}}$ and $\phi_{\mathbf{0i}}$ are the vectors of the eigenvector components that are part of the response calculation.

This expression gives us, for each mode and each node, an approximation of the horizontal component of the eigenvector using a quadratic function of the variation of the natural frequency. This method is preferable to using the sensitivity equations derived from Taylor developments and giving frequencies and eigenvector variations related to the design parameters. In fact, the approximate models used are a composition of these two expressions.

Once these approximate models are integrated in the MATLAB® calculation by modal superposition, it is possible to carry out the Monte Carlo simulations to find the optimal set of frequencies. This set is the one minimizing the value of $\mu_{4000} + 3.\sigma_{4000}$, where μ_{4000} is the mean of the vibration level at the payload/launch vehicle interface and σ_{4000} the standard deviation at this same point.

Thus, with 1000 sets of frequencies and 500 excitation cases for each, the optimal frequencies for the six selected modes are obtained (in hertz): 1.4947, 8.7690. 15.0026, 17.8008, 24.4733, and 33.9196.

These frequencies are assumed to lead to an average level at the payload/launch vehicle interface equal to 179.25 $\mathrm{m} \cdot \mathrm{s}^{-2}$ and a standard deviation equal to 7.59 $\mathrm{m} \cdot \mathrm{s}^{-2}$.

E. Going Back to the Geometry

The issue is now to find the values of the design parameters that allow the design of a structural configuration for which the natural frequencies are the ones previously computed. There are two

ways to achieve this step of the method, using two different tools: Monte Carlo simulations or a genetic algorithm.

1. Monte Carlo Simulations

In this case, a random sampling of the design parameters is made in the search space and, for each set of design parameters, a Nastran modal simulation is performed. In the given case, 10,000 Nastran modal simulations were performed, a high number of simulations sufficient to study the entire search space. The sampling is randomly implemented, according to a uniform law in the search space (parameters A, B, and C between 0.15 and 0.25 m and D between 0.07 and 0.13 m).

Thus, individuals (which correspond to structures) are obtained and located in a distance–mass space; a distance is defined from the optimal set of frequencies and calculated according to the following expression:

$$d = \sqrt{\sum_{i=1}^{6} (f_i - f_{i\,\mathrm{opt}})^2} \tag{18}$$

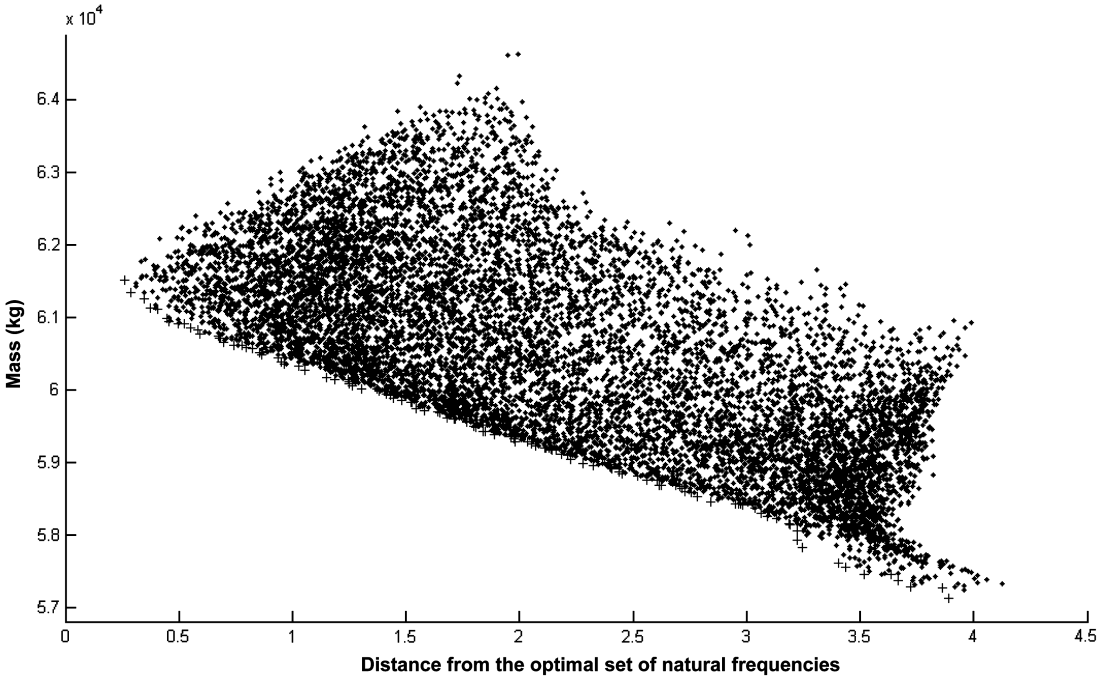

Fig. 8 Individuals from the Monte Carlo simulations located in the distance–mass space.

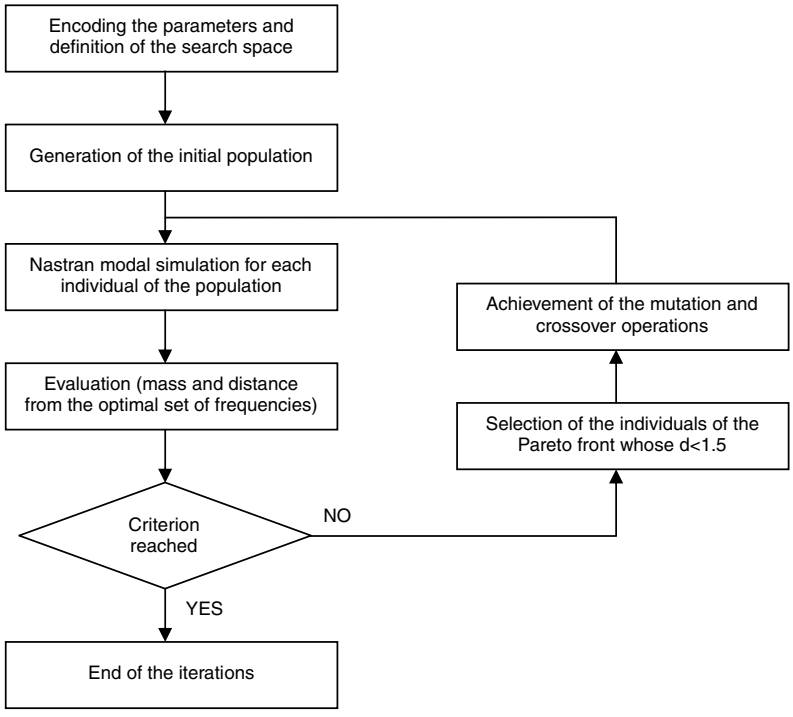

Fig. 9 Architecture of the genetic algorithm.

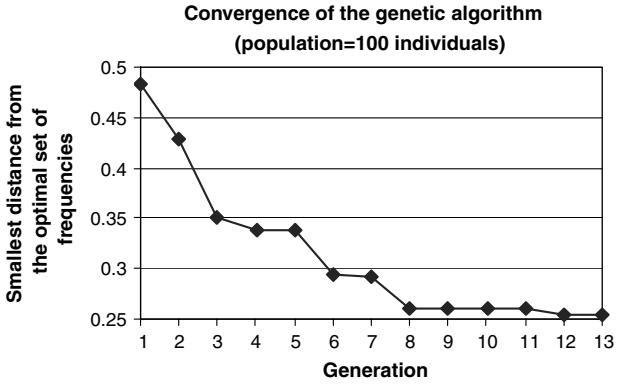

Fig. 10 Convergence of the genetic algorithm.

This graph is shown in Fig. 8. The Pareto front, which is symbolized by crosses in the figure, is selected; the individuals of this set are those that are not dominated. In this case, the Pareto front consists of 116 individuals; the smallest distance for an individual is 0.2600.

2. Genetic Algorithm

Another method that can be used to save computer time is genetic algorithms. The main principles of these algorithms are described in the Introduction.

The architecture of the algorithm is shown in Fig. 9. For example, the fitness function is replaced by the inclusion of the Pareto front in the distance–mass graph. A condition on the distance is also added; in fact, the individuals for which the distance is too high are highly unlikely to correspond to structures for which the behavior is close to the one sought.

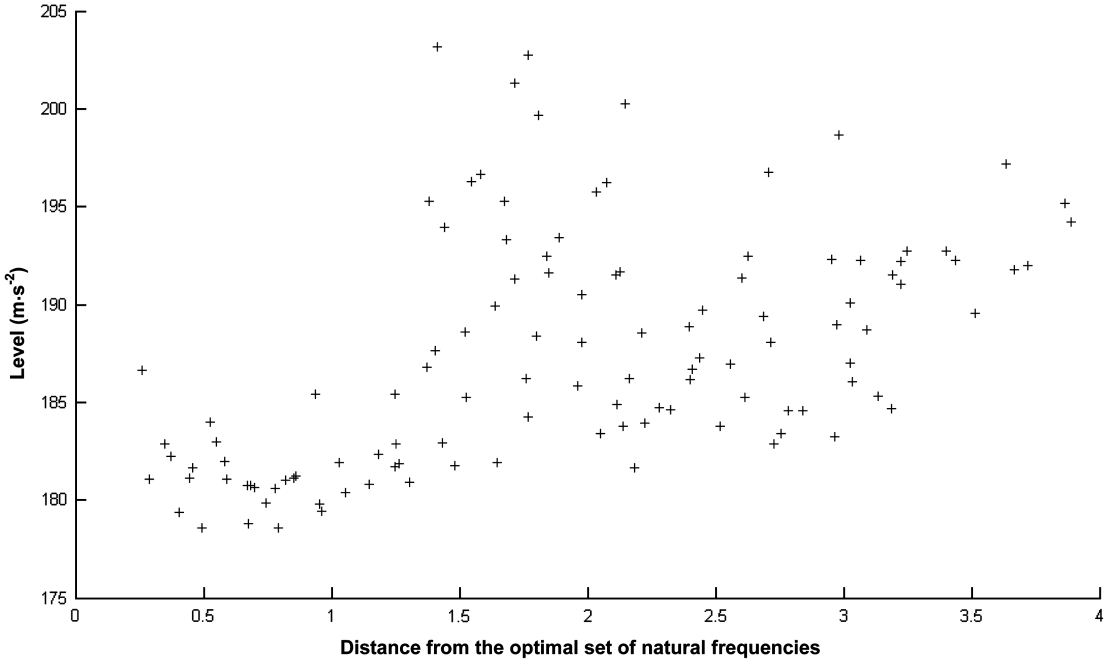

Fig. 11 Individuals from the Monte Carlo simulations located in the distance–vibration level space.

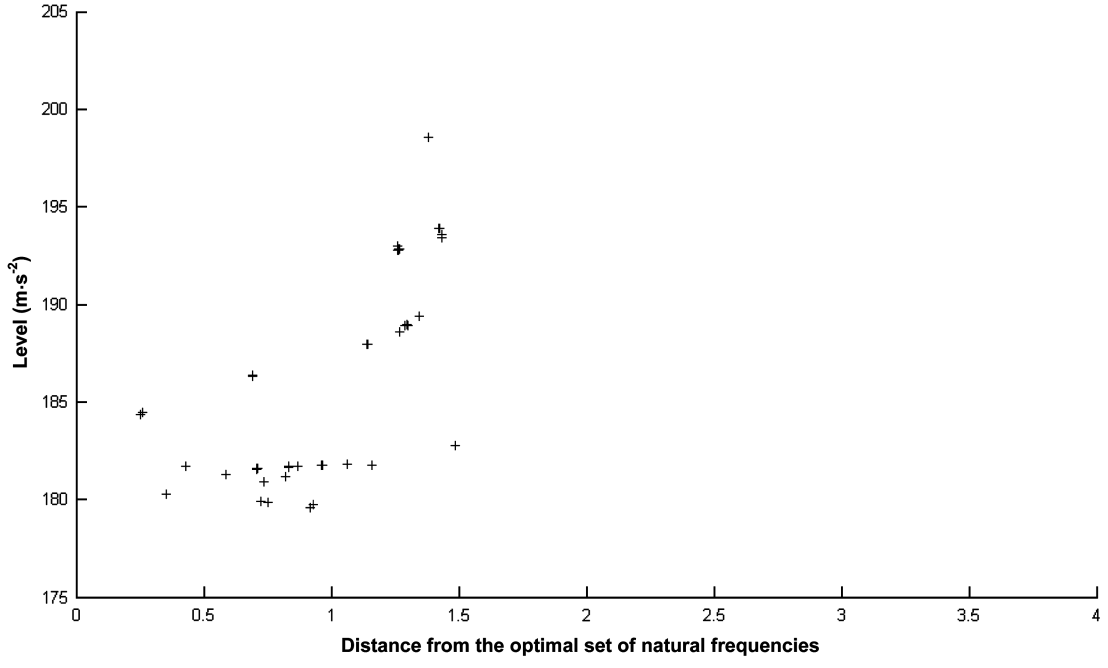

Fig. 12 Individuals from the genetic algorithm located in the distance–vibration level space.

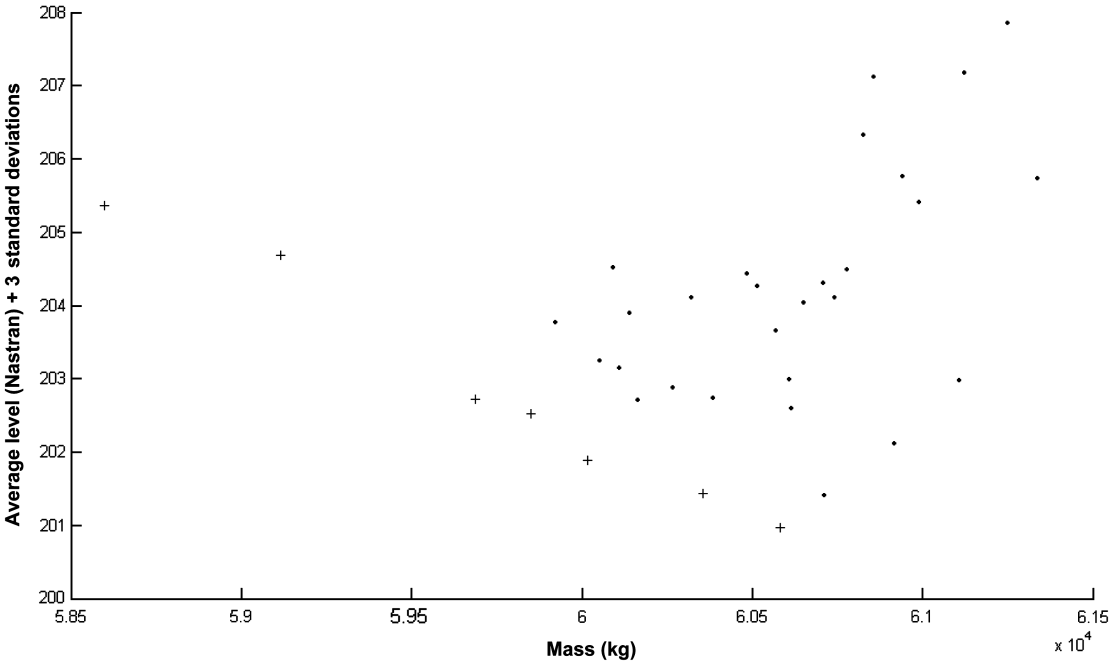

Fig. 13 Individuals from the Monte Carlo simulations located in the distance–objective function space.

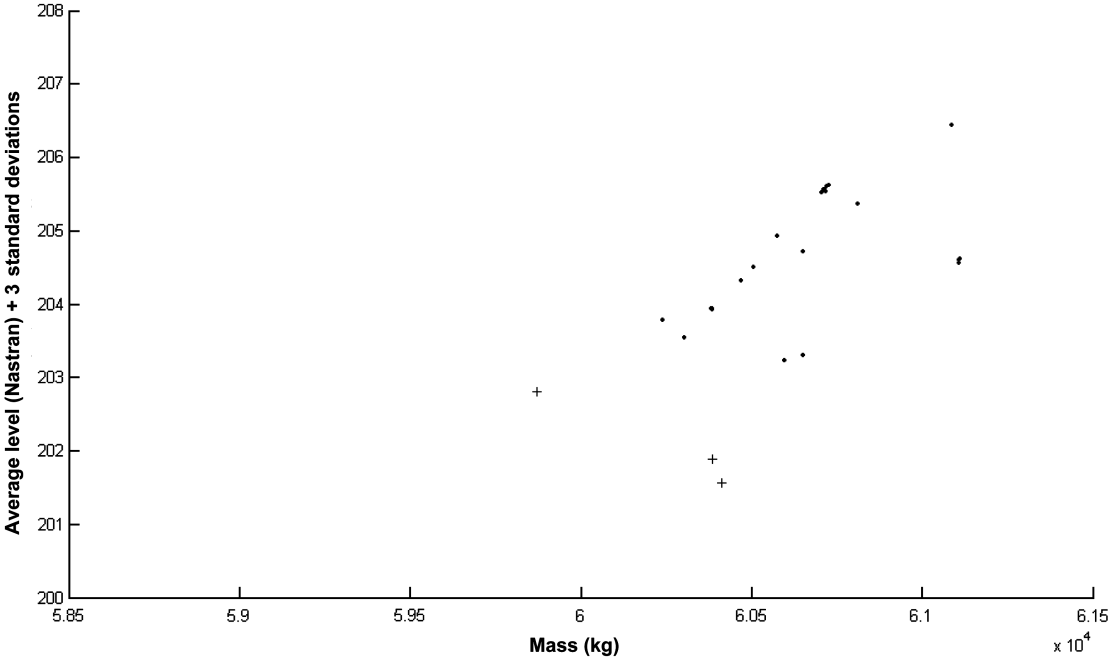

Fig. 14 Individuals from the genetic algorithm located in the distance–objective function space.

Working with the genetic algorithm parameters, different values can be adjusted: mutation and crossover probabilities (by bit) p_m and p_c, size of the population, criterion of termination, etc. The best results have been obtained while exploring the search space both locally and globally. In practice, once the selection is made, the individual for which the distance is the smallest will be mutated with a low probability ($p_m = 0.015$) to look for a better optimum in his neighborhood, whereas the other individuals selected are mutated with a high probability ($p_m = 0.5$) to explore the entire space and not converge toward a local optimum. Then, two-point crossovers are carried out to complete the population.

This implementation of the genetic algorithm provides very good results in our case. The convergence graph of a case with a population of 100 individuals is shown in Fig. 10. Obviously, the distance from the optimal set of frequencies of the individual for which distance is

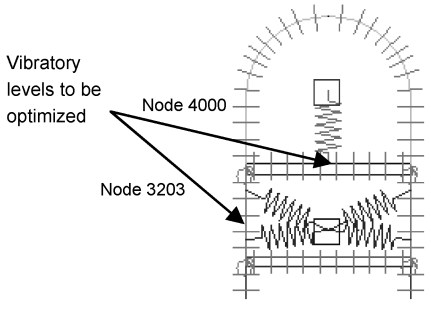

Fig. 15 Optimization of two points of the structure.

the smallest converges very quickly. After 12 iterations, an individual for which the distance is 0.2537 is obtained; this distance is smaller than the smallest distance computed in the 10,000 Monte Carlo simulations (that is, 0.2600).

3. Comparison of the Two Methods

Going back to the geometry, the main difference between the two methods is the computational time. In our case, the genetic algorithm is particularly efficient because a population of 100 individuals is sufficient to provide very satisfactory results; only 534 MATLAB® calculations were performed to reach convergence of the algorithm, whereas 10,000 Monte Carlo simulations were performed to obtain an individual for which the distance is almost as small as the one provided by the genetic algorithm. However, the convergence of the algorithm would be much slower and a larger population would be necessary in the case of a real industrial structure.

Furthermore, the Monte Carlo simulations provide more individuals than the genetic algorithm and so the final Pareto front will be composed of more individuals. This would give the designer a larger range of individuals from which to choose.

F. Checking the Computed Individuals

Finally, it is necessary to check that the individuals computed with the Monte Carlo simulations or the genetic algorithm satisfy the specifications of the vibration level at the payload/launch vehicle interface (mean and standard deviation).

For each individual of the last Pareto front, a Nastran modal transient simulation with the nominal excitation is performed to

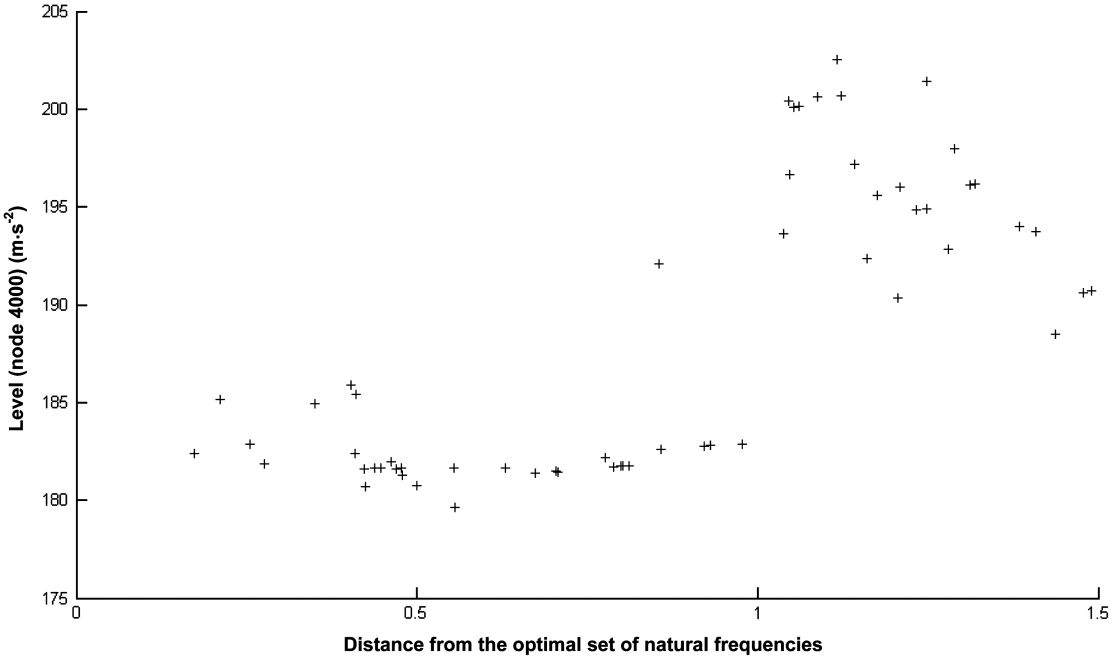

Fig. 16 Vibration level at the payload/launch vehicle interface in relation to the distance from the optimal set of natural frequencies.

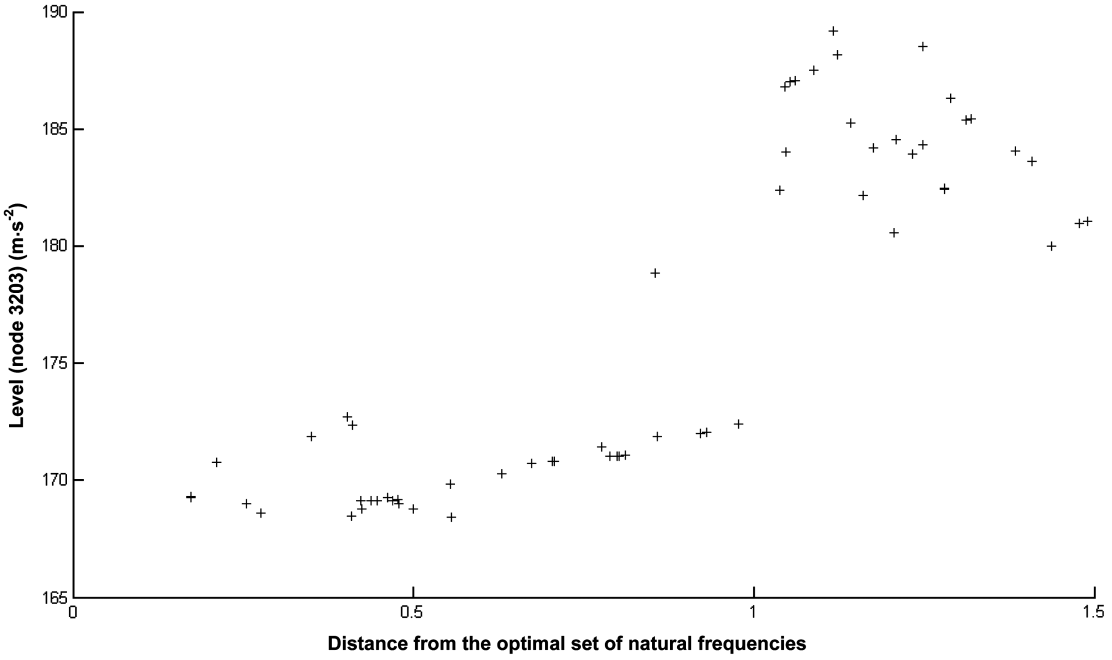

Fig. 17 Vibration level of the electronic equipment in relation to the distance from the optimal set of natural frequencies.

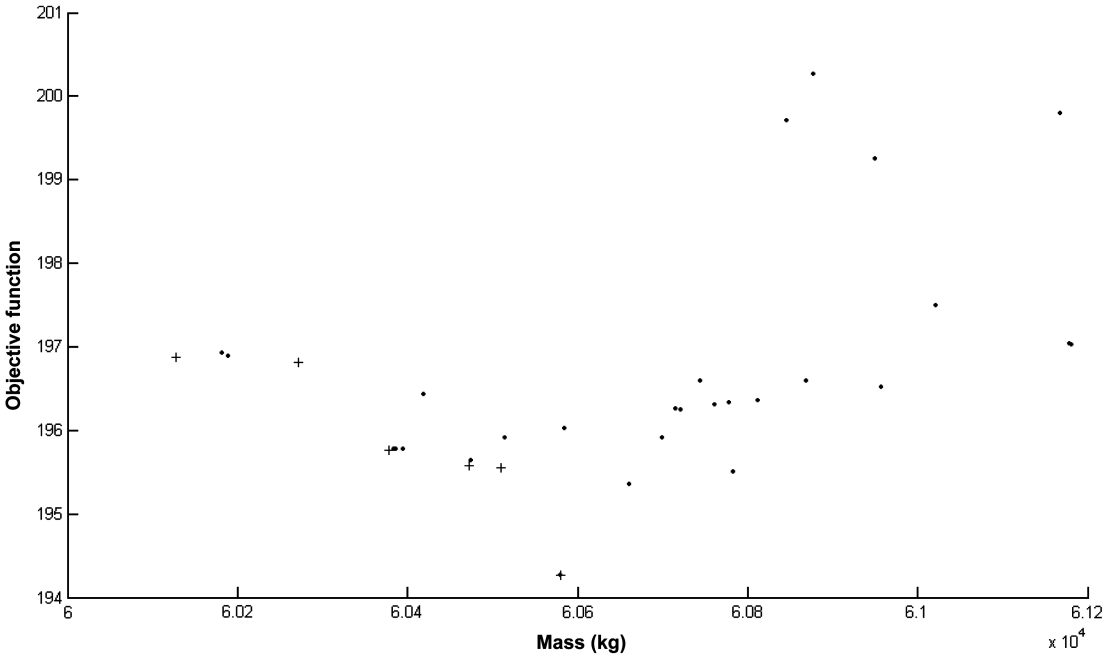

Fig. 18 Computed individuals located in the distance–objective function space.

compute the vibration level at the base of the payload and to check, for the individuals for which the distances are the smallest, that this level is close to the expected mean level (179.25 $m \cdot s^{-2}$). The results shown for the Monte Carlo simulations in Fig. 11 and for the genetic algorithm in Fig. 12 confirm that this is what happens; when the distance is smaller than 1.2, most individuals have a vibration level close to 179.25 $m \cdot s^{-2}$.

Then, among this population, the individuals whose levels are close to the one expected are selected to check that the standard deviation of these levels are also close to the expected value (7.59 $m \cdot s^{-2}$). This is performed by a MATLAB® calculation because of the way the uncertainty on the excitation (on its Fourier coefficient) is introduced; it would be impossible to implement the 500 different excitation cases with Nastran because the excitation must be given as a table with the magnitude for each step of time. To perform these simulations, the exact values of the components of the eigenvectors playing a part in the response of the structure are implemented; in fact, they have been calculated with Nastran to estimate the natural frequencies of each individual. Thus, the calculations are performed and the standard deviations computed are very close to the expected value.

After that, the individuals can be located in a mass-objective function space because they are the two values to be reduced. These graphs are shown for the Monte Carlo simulations in Fig. 13 and for the genetic algorithm in Fig. 14. The Pareto fronts, that is, the best individuals, are symbolized by crosses on the graphs; finally, the designer will choose from among these individuals the one he believes will give the best performance.

G. Robust Optimization of Several Points of the Structure

This method also makes it possible to tackle the issue of optimization at different points of the structure. For example, it could be necessary to minimize the vibration level and its sensitivity at two points of the launcher: the base of the payload and at the mounting points of electronic equipment, which cannot withstand high vibration levels (see Fig. 15).

The procedure to achieve this optimization is the same as the one presented earlier. Only a few steps are modified. First, it is necessary to implement in the MATLAB® script the calculation of the response at the new node being considered (node 3203). The formulation of the optimization problem is modified; the new objective function can be

$$\alpha(\mu_{3203} + k.\sigma_{3203}) + (1 - \alpha)(\mu_{4000} + k.\sigma_{4000}) \qquad (19)$$

where α is a constant chosen by the designer in accordance with the point of the structure under consideration (in this case, $\alpha = 0.5$). Finally, the designer must check the computed individuals in the two considered nodes.

Thus, the procedure has been completed. The Monte Carlo simulations on the natural frequencies provide the following six optimal natural frequencies (in hertz): 1.4839, 8.4099, 15.0142, 17.8683, 24.2813, and 34.1710. With these natural frequencies, the expected values of the vibration levels and their standard deviations are 183.02 and 8.00 $m \cdot s^{-2}$ at the base of the payload and 170.48 and 6.41 $m \cdot s^{-2}$ for the electronic equipment. It can clearly be noted that the values at the base of the payload are not as satisfactory as those obtained when the optimization is achieved considering only node 4000.

Then, the genetic algorithm is used to obtain individuals for which the mass and distance from the new optimal set of natural frequencies are as low as possible. After that, the computed individuals are tested to check that they have the required behavior concerning the vibration level at the base of the payload (mean and standard deviation). A Nastran modal transient simulation with the nominal excitation makes it possible to compute the vibration level at the base of the payload (Fig. 16) and at electronic equipment mounting points (Fig. 17). As the selected individuals are those with the smallest distances, the computed levels are close to the expected mean levels. Then, among this population, the individuals for which the levels are close to the ones expected are selected to check by means of MATLAB® calculations that the standard deviations of these levels are also close to the expected values. Finally, the individuals are located in a mass-objective function space (Fig. 18) and the designer will choose from among these individuals.

VI. Conclusions

This paper proposes the FDM, a robust optimization procedure in which the optimization is primarily performed on the natural modes of the structure before going back to the geometry using the FE model. This approach to the issue, inspired by the methodological framework of systems engineering, has several advantages.

First, the MATLAB® calculation of the response by modal superposition allows the designer to avoid performing numerous Nastran simulations, which are costly in computer time. It allows an

optimization of the natural frequencies using Monte Carlo simulations. Furthermore, the use of genetic optimization allows the designer to explore a larger search space than would a classic gradient-based optimization procedure, even though he must set limits. It is thus possible to design the geometry of a structure for which the natural modes are those provided by the optimization procedure. Thus, the FDM makes it possible to define structures that the classic method cannot offer the designer. Taguchi's approach, for example, would not provide an accurate solution for industrial design problems that involve highly nonlinear behavior, because the responses of such systems would not be accurately approximated by the polynomial functions of the design factors; even if the system's behavior is almost linear, it only allows the designer to find a structural configuration close to the nominal one and, therefore, achieve a local robust optimization. Moreover, the method for integrating uncertainties about excitation in the FDM is much more "in line with the physics" because the energy of the signal is kept constant, with variation spaces of the Fourier coefficients depending on the frequency. Thus, this method allows for the design of structural configurations for which the behavior is very satisfying as far as the mass, the vibration level at the payload/launch vehicle interface, and its sensitivity are concerned.

Furthermore, the proposed alternative method for robust optimization can be particularly useful at the very beginning of a project, during the preliminary design. More precisely, if the mechanical system's geometry is lightly modified (that is what often happens in the first stages of a design), for example, by adding a stiffness, the design parameters will consequently be modified but the optimum found in the FDM's third step will not change. What is more, FDM permits setting an optimum (in terms of natural modes) independently of other constraints (relating to the geometry or manufacturing process, for example), which could be modified later. In both cases, it will not be necessary to reperform the whole procedure but only the fourth step.

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